The results presented also emphasize the importance of low values of $\rho k/c$ in order to achieve minimum insulation weight.

Boundary Conditions at the Outer Edge of the Boundary Layer on Blunted Conical Bodies

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In most boundary layer analyses for blunted conical bodies the entropy gradient produced by the curved shock usually is neglected. However, Refs. 1 and 2 have pointed out that the boundary layer characteristics may be affected appreciably by this entropy variation, which influences the development of the boundary layer in two different ways. First, it causes a continuous change of the flow properties at the edge of the boundary layer in the streamwise direction, and second, it produces a velocity gradient at the outer edge in the normal direction, i.e., $(\partial u/\partial y)_e \neq 0$. In this investigation, only the streamwise variation at the edge of the boundary layer will be considered; thus, by assuming that $(\partial u/\partial y)_e \simeq 0$, the local similarity concept can be applied.

The method developed herein is based on mass flow conservation inside the boundary layer and provides a convenient means of determining the variation of the flow properties at the outer edge of the boundary layer due to the local entropy gradient. Assuming local similarity, the mass flow equation as given by Lees[3] can be applied for the laminar portion of the boundary layer. The external conditions at the edge of the boundary layer then can be determined by comparing the mass flow imbedded in the boundary layer with the mass flow in front of the shock entering at freestream conditions.

For a thermally and calorically perfect gas, the mass flow inside the boundary layer is

$$\dot{\omega} = 2^{3/2}\pi\left[(h_{01})^{1/2}\rho_{01}\mu_{01}R_0^3\int_0^{\bar{s}} \bar{u}_e\bar{\mu}_e\bar{\rho}_e\bar{r}^2 d\bar{s}\right]^{1/2} f(\eta_e) \qquad (1)$$

The mass flow in front of the shock at freestream conditions may be expressed as

$$\dot{m} = \rho_\infty u_\infty \pi \bar{y}^2 R_0^2 \equiv \dot{\omega} \qquad (2)$$

If Eqs. (1) and (2) are used to eliminate $\dot{\omega}$, there results an equation for the normalized shock coordinate $\bar{y}$, corresponding to a station $\bar{s}$, namely,

$$\bar{y} = ZR_0^{-1/4}\left(\int_0^{\bar{s}} \bar{u}_e\bar{\mu}_e\bar{\rho}_e\bar{r}^2 d\bar{s}\right)^{1/2} \qquad (3)$$

where

$$Z = \left\{2^{3/2} f(\eta_e)\,\frac{[(h_{01})^{1/2}\rho_{01}\mu_{01}]^{1/2}}{\rho_\infty u_\infty}\right\}^{1/2} \qquad (4)$$

Equations (3) and (4) lead to a similarity parameter $\bar{s}'/Re_s^{1/4}$,

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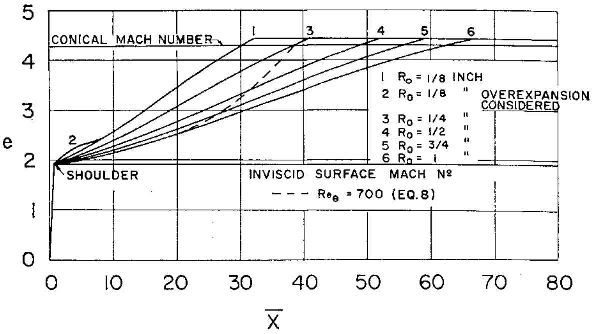

Fig. 1 Mach number distribution on blunt conical bodies; $M_\infty = 7.9$, $Re_\infty = 2.02 \times 10^6$ ft, $T_0 = 1660°$R, $P_0 = 600$ psi, $\theta_b = 20°$

as given in Ref. 2, where $\bar{s}'$ is the normalized coordinate measured from the apex of the cone and Re_s is the local Reynolds number, based on the normal shock stagnation conditions; $Re_s = [\rho_{01}(h_{01})^{1/2}/\mu_{01}]\bar{s}'$. If this parameter is introduced, the dependence of the external flow properties on the nose radius vanishes.

When the shock shape in the form of an analytic expression $\bar{y} = f(\bar{x})$ and the distribution of the static pressure on the surface of the body are known, the local stagnation pressure at the edge of the boundary layer can be determined by solving the integral in Eq. (3) in a stepwise procedure. The local velocity $\bar{u}_e$, viscosity $\bar{\mu}_e$, and density $\bar{\rho}_e$, normalized with respect to the normal shock stagnation conditions, are given by the ratio of the local static-to-stagnation pressure. The static pressure in the nose region may be approximated by the modified Newtonian distribution, whereas on the conical portion of the body the static pressure is well represented by the pointed cone value. The observed overexpansion behind the shoulder on a blunted cone has only a little influence at high freestream Reynolds numbers but becomes more important when Re_∞ decreases.

Figure 1 presents the results of an application of this method; shown is the variation of the local Mach number external to the boundary layer on the conical portion of a spherically capped cone for various nose radii R_0 and for constant unit Reynolds number in the freestream, plotted vs the axial coordinate $\bar{x}$.

The results shown in Fig. 1 plus additional results given in Ref. 5 for a variety of flow conditions and nose radii indicate that the Mach number external to the boundary layer increases almost linearly until it finally approaches the value given by the surface Mach number of the pointed cone. With this approximation, Eq. (3) can be integrated to yield an approximate expression for the swallowing distance, i.e., the distance in which the external Mach number increases to the conical value, in terms of the flow parameters and the cone half angle. This expression may be found as follows. Simplifying the equation for the contour of the body $\bar{r} = \cos\theta_b + (\bar{s} - \bar{s}_s)\sin\theta_b$ to

$$\bar{r} = \bar{s}\sin\theta_b \qquad (5)$$

and assuming a viscosity variation

$$\mu_e = \lambda T_e^{1/2} \qquad (6)$$

where λ is introduced in order to match Sutherland's equation, one obtains for $\bar{s}_c$

$$\bar{s}_c \cong \left[\frac{3}{2}\,\frac{(R/\gamma)^{1/2}}{\lambda p_c}\,\frac{\rho_\infty{}^2 u_\infty{}^2}{(3M_c + M_s)}\,\frac{\bar{y}_c{}^4 R_0}{f^2(\eta_e)\sin\theta_b}\right]^{1/3} \qquad (7)$$

where the integration is performed in the limits

$$\bar{s} = 0 \qquad M_e = M_s$$
$$\bar{s} = \bar{s}_c \qquad M_e = M_c$$

A comparison between the foregoing approximation and the

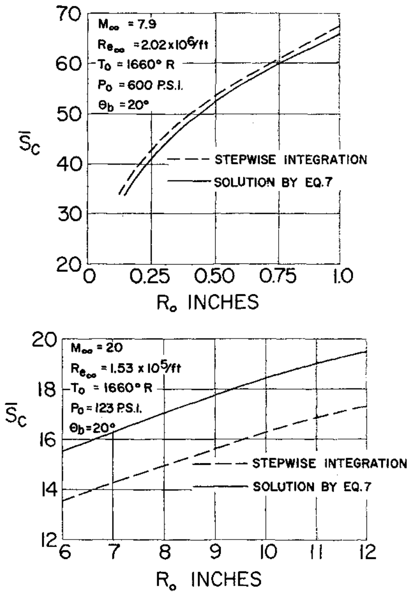

Fig. 2 Swallowing distance $\bar{s}_c$ as a function of the nose radius

stepwise integration is presented in Fig. 2. It is seen in the first example that the results from Eq. (7) are in good agreement with those obtained by stepwise integration of Eq. (3). However, in the second case, where the freestream Mach number is comparatively low, the difference is of the order of 15%.

By means of the linearized relation for $\bar{s}_c$, it also is possible to make an estimate of the transition point when the transition Reynolds number, based on the momentum thickness, is known:

$$Re_\theta \cong \left\{\frac{p_c}{6\lambda\,(R/\lambda)^{1/2}T_0}\left[3(M_c - M_s)\,\frac{\bar{s}}{\bar{s}_c} + M_s\right]\times \left(1 + \frac{\gamma - 1}{2}\,M_e^2\right) R_0\bar{s}\right\}^{1/2}\int_0^{\eta_e} f'(1 - f')d\eta \quad (8)$$

The transition Reynolds number predicted by Eq. (8) is in fairly good agreement with the experimental data of Ref. 4. The numerical values are listed in Table 1.

In addition to this comparison, the transition points also were calculated for the conditions as given in Fig. 1, assuming that transition occurs when $Re_\theta = 700$. This result has been obtained for a blunted cone with $R_0 = 0.2$ and $\theta = 15°$ under nearly the same freestream conditions as are presented in Ref. 2.

As mentioned in the foregoing, the dependence of the external flow properties on the nose radius vanishes when they are expressed in terms of the similarity parameter of Ref. 2. This is shown in Fig. 3, where the Mach number variation obtained by the present analysis is compared to the experimental data of Ref. 4.

Table 1

Experimental data (Ref. 4)			Present analysis	
R_0, in.	$\bar{x}_{\rm trans}$	$Re_{\theta{\rm trans}}$	$Re_{\theta{\rm trans}}$	$M_{e{\rm trans}}$
0.0480	115	620	660	3.30
0.0830	55	550	490	2.71
0.1650	39	?	460	2.58
0.4900	12	?	385	2.35

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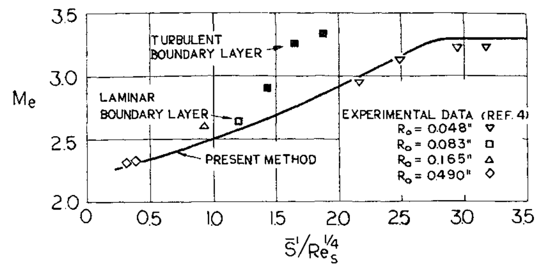

Fig. 3 Similarity parameter for similar blunt bodies; $M_\infty = 3.81$, $T_0 = 560°$R, $\theta_b = 7.5°$

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Integral Method Solutions of Laminar Viscous Free-Mixing

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Introduction

THE integral method furnishes a convenient means for the study of nonsimilar boundary-layer problems such as in viscous free-mixing. This paper deals with laminar, two-dimensional, symmetric and axisymmetric, incompressible, uniform pressure wakes and jets. Solutions are derived by using the simple one-strip integral method and are presented in closed form. It is shown that the present theory agrees reasonably well with other more accurate, but very cumbersome, methods of solution. Compressibility, turbulence, thermal, and other diffusive properties can be studied by analogous means.

Analysis

The following boundary-layer equations are assumed to govern the viscous free-mixing previously discussed and represented schematically in Fig. 1:

Continuity

$$u_x + v_y + \epsilon(v/y) = 0 \quad (1)$$

Momentum

$$uu_x + vu_y = \nu[u_{yy} + \epsilon(u_y/y)] \qquad p = \text{const} \quad (2)$$

where $\epsilon = 0, 1$ for two-dimensional and axisymmetric flow, respectively; x and y are the streamwise and normal coordinates with velocity components u and v; p denotes pressure; ν kinematic viscosity; and subscripts x and y denote partial differentiation with respect to the indicated variable.

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